Questions From Lecture 2

Tuesday, January 23, 2018 1:21 PM

Saood Almarzooqi
MTH213 Spring 2018

* Q. $\gcd(104, 54) = 2$

$$\begin{array}{r} 1 \\ 54 \overline{)104} \\ -54 \\ \hline 50 \end{array} \longrightarrow \begin{array}{r} 1 \\ 50 \overline{)54} \\ -50 \\ \hline 4 \end{array} \longrightarrow \begin{array}{r} 12 \\ 4 \overline{)50} \\ -48 \\ \hline 2 \end{array} \longrightarrow \begin{array}{r} 2 \\ 2 \overline{)4} \\ -4 \\ \hline 0 \end{array}$$

* Find prime factorization of 308

$$308 = 2 \times 154 = 2^2 \times 77 = \underline{2^2 \times 11 \times 7}$$

* Solve over $\mathbb{Z}_6$ & fin # of solutions

① $2x = 4$

$x = 2, x = 5$

② - $4x = 22$

$2x = 4$

$x = 2, x = 5$

③ - $4x = 5$

$2x = 5$

no solution

* ① - $23 \bmod 19 = 19 - (23 \bmod 19) = 15$

$$-40 \bmod 27 = 27 - (40 \bmod 27) = 14$$

Questions From Lecture 3

Saturday, January 27, 2018 10:24 PM

aS Saood Almarzooqi

Q1. solve over $\mathbb{Z}_{20}$

$$4x = 12$$

four solutions

$\gcd(4,20) = \underline{4}$, $4 \mid 12$ ✓

$x = 3$, $x = 8$, $x = 13$,

$x = 18$

Q2. Describe all integers, say x, such that

$$4x \equiv 12 \pmod{20}$$

by previous solution, there are 4 solutions:

$x = 3 + 20k$, $k \in \mathbb{Z}$

$x = 8 + 20k$, $k \in \mathbb{Z}$

$$x = 13 + 20k, \ k \in \mathbb{Z}$$

$$x = 18 + 20k, \ k \in \mathbb{Z}$$

These solutions, however, can be simplified to one:

$$x = 3 + 5(k + 4l), \ k \in \mathbb{Z}, \text{ and } l \in \mathbb{Z}$$

Q3. Find $(5661)_7 + (3446)_7$

$$\begin{array}{r} \overset{1}{5}\overset{1}{6}\overset{1}{6}1 \\ (5661)_7 \\ + (3446)_7 \\ \hline (12440)_7 \end{array}$$

Find $(7372)_8 - (1427)_8$

$$\begin{array}{r} (73\overset{6}{\cancel{7}}2)_8 \\ - (1427)_8 \\ \hline (6743)_8 \end{array}$$

$$(6743)_8$$

Find $(321)_5 \times (43)_5$

$$
\begin{array}{r}
(321)_5 \\
\times\ (43)_5 \\
\hline
(43)_5 \\
(110)_5 \\
(1300)_5 \\
(1400)_5 \\
+\ (22000)_5 \\
\hline
(30403)_5
\end{array}
$$

Saood Almarzooqi

Q1. x = # of defective computers

$x \equiv 5 \bmod \underbrace{9}_{\to m_1}$

$x \equiv 10 \bmod \underbrace{22}_{\to m_2}$

$x \equiv 4 \bmod \underbrace{5}_{\to m_3}$

$0 \leq x < 990$

$\gcd(9, 22) = 1$

$\gcd(9, 5) = 1$

$\gcd(22, 5) = 1$

by CRT, there $\exists$ one unique solution $\in 0 \leq x < 990$

$d_1 = (m_2 \, m_3)^{-1} \bmod m_1 = (110)^{-1} \bmod 9 = 5$

$d_2 = (m_1 \, m_3)^{-1} \bmod m_2 = (45)^{-1} \bmod 22 = 1$

$d_3 = (m_1 \, m_2)^{-1} \bmod m_3 = (198)^{-1} \bmod 5 = 2$

$x = [(m_2 \, m_3) d_1 \, r_1 + (m_1 \, m_3) d_2 \, r_2 + (m_1 \, m_2) d_3 \, r_3] \bmod (m_1 m_2 m_3)$

$$= [110(5)(5) + 45(10) + 198(2)(4)] \bmod 990 = 824$$

Q2. solve for x, $0 \le x \le 260$

$$x \equiv \underbrace{19}_{\to r_1} \bmod \underbrace{20}_{\to m_1}$$

$$x \equiv \underbrace{12}_{\to r_2} \bmod \underbrace{13}_{\to m_2}$$

$$\gcd(13, 20) = 1$$

by CRT, there $\exists$ one unique solution $\in 0 \le x < 260$

$$d_1 = m_2^{-1} \bmod m_1 = (13)^{-1} \bmod 20 = 7$$

$$d_1 = m_2^{-1} \bmod m_1 = (13)^{-1} \bmod 20 = 17$$

$$d_2 = m_1^{-1} \bmod m_2 = (20)^{-1} \bmod 13 = 2$$

$$x = [13(17)(19) + 20(2)(12)] \bmod 260 = 259$$

Questions from Lecture 4

Monday, January 29, 2018 4:40 AM

Saood Almarzooqi

Q1. x = # of defective computers

$x \equiv 5 \bmod 9 \rightarrow m_1$

$x \equiv 10 \bmod 22 \rightarrow m_2$

$x \equiv 4 \bmod 5 \rightarrow m_3$

$0 \leq x < 990$

$\gcd(9, 22) = 1$

$\gcd(9, 5) = 1$

$\gcd(22, 5) = 1$

by CRT, there $\exists$ one unique solution $\in 0 \le x < 990$

$d_1 = (m_2 m_3)^{-1} \bmod m_1 = (110)^{-1} \bmod 9 = 5$

$d_2 = (m_1 m_3)^{-1} \bmod m_2 = (45)^{-1} \bmod 22 = 1$

$d_3 = (m_1 m_2)^{-1} \bmod m_3 = (198)^{-1} \bmod 5 = 2$

$x = [(m_2 m_3) d_1 r_1 + (m_1 m_3) d_2 r_2 + (m_1 m_2) d_3 r_3] \bmod (m_1 m_2 m_3)$

$$= [110(5)(5) + 45(10) + 198(2)(4)] \bmod 990 = 824$$

Q2. solve for x, $\quad 0 \le x \le 260$

$$x \equiv \overset{r_1}{19} \bmod \overset{m_1}{20}$$

$$x \equiv \overset{r_2}{12} \bmod \overset{m_2}{13}$$

$$\gcd(13, 20) = 1$$

by CRT, there $\exists$ one unique solution $\in 0 \le x < 260$

$$d_1 = m_2^{-1} \bmod m_1 = (13)^{-1} \bmod 20 = 7$$

$$d_1 = m_2^{-1} \bmod m_1 = (13)^{-1} \bmod 20 = 17$$

$$d_2 = m_1^{-1} \bmod m_2 = (20)^{-1} \bmod 13 = 2$$

$$x = [13(17)(19) + 20(2)(12)] \bmod 260 = 259$$

Muhammed Dada HW 4

H/W 4

Q1) Find $\gcd(37, 44)$. Then find k_1, k_2 s.t. $\gcd(37,44) = 37k_1 + 44k_2$.

Q2) $\text{Lcm}[112, 56]$.

Q1) $\gcd(37, 44)$

$$37 \overline{)44} \quad 1,\ -37,\ 7 \qquad 7\overline{)37} \quad 5,\ -35,\ 2 \qquad 2\overline{)7} \quad 3,\ -6,\ 1 \qquad 1\overline{)2} \quad 2,\ -2,\ 0$$

$1 = 7 - 2 \times 3$

$1 = 7 - 3(37 - 7 \times 5) = -3 \times 37 + 7 \times 16$

$1 = -3 \times 37 + 16(44 - 37 \times 1)$

$1 = 16 \times 44 - 16 \times 37 - 3 \times 37$

$1 = 16 \times 44 - 19 \times 37$

$k_1 = -19$

$k_2 = 16$

Q2) $\text{Lcm}[112, 56] = \dfrac{112 \times 56}{\gcd(112, 56)} = \dfrac{112 \times \cancel{56}}{\cancel{56}} = 112.$

$$56 \overline{)112} \quad 2,\ -112,\ 0$$

HW5

Tuesday, February 6, 2018 6:24 PM

Saood Al-Marzooqi

QUESTION 1. (i) Let n, m be distinct prime positive integers. Convince me that there exist $c_1, c_2 \in Z$ such that $2018 = nc_1 + mc_2$ (direct proof)

(ii) Let $m, a, b \in Z$. Assume that $m \mid a$ and $m \mid b$. Convince me that for every integers i_1, i_2, we have $m \mid (ai_1 + bi_2)$. (i.e., show that $ai_1 + bi_2 = mk$ for some $k \in Z$). (direct proof)

i) if n, m are prime (has two factors: 1 and itself) and $n \neq m$,

then $\gcd(n, m) = 1$. Thus, there are alway $a_1, a_2 \in Z$

where $1 = n\,a_1 + m\,a_2$. If we multiply both sides with

2018, we get $2018 = \overbrace{2018\,a_1}^{c_1}\,n + \overbrace{2018\,a_2}^{c_2}\,n$. we set $c_1 = 2018a_1$.

and $c_2 = 2018\,a_2$; Thus proving $\underline{2018 = c_1\,n + c_2\,m}$

ii) if $m \mid a$, then there is $\exists$ $n_a \in Z$ such that

$n_a = \frac{a}{m}$. Same with $b \rightarrow n_b = \frac{b}{m}$

$i_1 n_a + i_2 n_b = k$ is true <u>iff</u> k is a multiple of $\gcd(i_1, i_2)$

$$\left(i_1 \frac{a}{m} + i_2 \frac{b}{m}\right)^{\times m} = k^{\times m}$$

$$i_1 a + i_2 b = mk$$

(iii) Let x, y be rational numbers. Prove that $x + y$ is rational (i.e., show that $x + y = c/d$ for some integers $c \in Z$ and $d \in Z^*$)

if x, y are rational, there must be $n_x, n_y, d_x, d_y \in Z$ such that

$$x = \frac{n_x}{d_x}, \; y = \frac{n_y}{d_y}.$$

$$x + y = \left(\frac{n_x}{d_x}\right)^{\times d_y}_{\times d_y} + \left(\frac{n_y}{d_y}\right)^{\times d_x}_{\times d_x} = \frac{d_y n_x + d_x n_y}{d_x d_y}$$

The sums and products of integers <u>must be</u> integers as well; thus,

$$\frac{d_y n_x + d_y n_y}{d_x d_y} = \frac{c}{d} \text{ for some } c, d \in Z$$

(iv) Assume $31 = ac_1 + bc_2$ and $gcd(a, b) \neq 1$. Find $gcd(a, b)$ (of course a, b in N^* and $c_1, c_2 \in Z$).

We know that $a c_1 + b c_2 = m \; gcd(a, b)$, $m \in Z$

Since 31 is a prime number and $\gcd(a, b) \neq 1$, then $\gcd(a, b) = 31$ and $m = 1$ (because 31 cannot the product of any two positive integers except 1 and 31)

(v) Convince me that $1963.2018\overline{2017}$ is a rational number.

Let $x = 1963.2018\overline{2017}$

$$999x \rightarrow \begin{array}{r} 19632018.2017 \\ -\quad 1963.2018 \\ \hline 19630054.9999 \end{array}$$

$$x = \frac{19630054.9999}{999} = \frac{196300549999}{9990000}$$

(vi) (optional) Choose a number $n \in N^*$. Convince me that there there is a positive integer k such that $k, k+1, k+2, k+3, ..., k+n$ are non-prime integers [If this is not beautiful, in my opinion "beauty" has no meaning!!... This result tells you choose $n = 2889898765432190$. Then there must exist a number k, so that the next n-consecutive numbers are not prime numbers. Remind me on Thursday to do this question in CLASS)

except for 1, 2, 3 (the only consequtive prime numbers),

after each prime each number there must be at least

one non-prime number. For example, all even numbers after

2 are non-prime (because they are divisible by 2), so each

prime number from 3 will be followed by an even number.

so $n \geq 1$ This is my humble attempt ☺

[1] Let a be an odd number. Show a^2 is odd number.

[2] Let a be an even integer. Show a^2 is even integer.

[3] Show $\sqrt{12}$ is irrational. (Hint: isolate the prime).

[4] Show that $\sqrt{45}$ is irrational. (Hint: see question above).

[5] Show $\sqrt{15}$ is irrational. (Try to use method 1).

[6] Irrational + irrational = irrational. True or false. If false, give counter example.

Solutions: [1] Let $a = 2m+1, \quad m \in \mathbb{Z}$

$$a^2 = (2m+1)^2$$
$$= 4m^2 + 4m + 1$$
$$= 2(2m^2 + 2m) + 1$$

let $2m^2 + 2m = k, \quad k \in \mathbb{Z}$

Hence, $a^2 = 2k + 1 \Rightarrow$ odd.

$\therefore$ square of an odd number is always odd.

[2] let $a = 2n, \quad n \in \mathbb{Z}$.

$$a^2 = (2n)^2 = 4n^2 = 2(2n^2).$$

let $2n^2 = w, \quad w \in \mathbb{Z}$

Hence, $a^2 = 2w \Rightarrow$ even.

$\therefore$ square of an even number is always even.

[3] ~~12~~ $\sqrt{12} = \sqrt{2 \times 2 \times 3} = 2\sqrt{3}$

(~~3 is a prime number, hence $\sqrt{3}$ is irrational.~~)

Deny. Say $2\sqrt{3}$ is a rational number.

Hence, $2\sqrt{3} = \dfrac{a}{b}, \quad a \in \mathbb{Z}, \ b \in \mathbb{Z}^*$

$$\sqrt{3} = \frac{a}{2b}$$

~~Squar~~ but $\sqrt{3}$ has been established as an irrational number,

irrational $\neq$ rational (contradiction).

Hence $2\sqrt{3}$ i.e $\sqrt{12}$ is an irrational number.

[4] $\sqrt{45} = \sqrt{5\times3\times3} = 3\sqrt{5}$

Deny. Say $3\sqrt{5}$ is rational.

$$3\sqrt{5} = \frac{a}{b}, \quad a,b \in \mathbb{Z},\ b \neq 0$$

$$\sqrt{5} = \frac{a}{3b}.$$

but $\sqrt{5}$ is an irrational number.

irrational $\neq$ rational. (contradiction)

Hence $3\sqrt{5}$ i.e $\sqrt{45}$ is an irrational number.

[5] Deny. Say $\sqrt{15}$ is ~~irrational~~ rational.

$$\sqrt{15} = \frac{a}{b}, \quad a,b \in \mathbb{Z},\ b \neq 0$$

Square both sides: $\quad 15 = \frac{a^2}{b^2}$

We have established that a and b are odd numbers.

So let $a = 2m+1,\ m \in \mathbb{Z}$

$b = 2k+1,\ k \in \mathbb{Z}$

$$15(4k^2+4k+1) = 4m^2+4m+1$$

$$60k^2+60k+15 = 4m^2+4m+1$$

$$\Rightarrow 60k^2+60k+14 = 4m^2+4m.$$

Divide by 4 $\Rightarrow$ $\underbrace{15k^2+15k+\frac{14}{4}}_{\text{not integer as } 4 \nmid 14} = \underbrace{m^2+m}_{\text{integer}}$

Hence $15k^2+15k+\frac{7}{2} \neq m^2+m$

So we ~~can~~ conclude $\sqrt{15}$ is not rational, i.e irrational.

[6] Irrational + irrational = ~~irrat~~ irrational is false.

Counter example ; $(\sqrt{2}+1)+(-\sqrt{2}) = 1, \quad 1 \in \mathbb{Q}\ \&\ 1 \in \mathbb{Z}$

Tasneem Batool

Homework questions:

[1] Let $n = 84$

i. Find all factors of 84.

ans:

1	84
2	42
3	28
4	21
6	14
7	12

ii. Say $d_1, d_2, \dots d_k$ are all factors of n. Find $\phi(d_i)$ for each $1 \le i \le k$.

ans: $d_1 = 1$ $\quad d_2 = 2$ $\quad d_3 = 3$ $\quad d_4 = 4 = 2^2$

$\phi(1) = 1.$ $\quad \phi(d_2) = 1$ $\quad \phi(3) = 2$ $\quad \phi(2^2) = 2$

$d_5 = 6 = 3 \times 2$ $\quad d_6 = 7$ $\quad d_7 = 12 = 2^2 \times 3$ $\quad d_8 = 14 = 7 \times 2$

$\phi(3 \times 2) = 2$ $\quad \phi(7) = 6$ $\quad \phi(d_7) = 4$ $\quad \phi(14) = 6$

$d_9 = 21 = 7 \times 3$ $\quad d_{10} = 28 = 2^2 \times 7$ $\quad d_{11} = 42 = 2 \times 21$ $\quad d_{12} = 84$

$\phi(21) = 12$ $\quad \phi(28) = 12$ $\quad \phi(42) = \phi(21) = 12$ $\quad \phi(d_{12}) = 24$

iii. Find $\phi(d_1) + \phi(d_2) + \dots + \phi(d_k)$

ans: $1 + 1 + 2 + 2 + 2 + 6 + 4 + 6 + 12 + 12 + 12 + 24$

$= 84 = n$

iv. Let $F = \{ 1 \le a < 88 \mid \gcd(a, 88) = 11 \}$. Find $|F|$.

ans: $n = 88$, $k = 11$.

Then $|F| = \phi\left(\frac{n}{k}\right) = \phi\left(\frac{88}{11}\right) = \phi(8) = 4.$

[2] i. Find $17^{41} \bmod 41$. Justify your answer. (note that $\phi(41) = 40$.) So $17^{40} = 1$ in planet $\mathbb{Z}_{41}$. Multiply both sides with 17, we get $17^{41} = 17$ in $\mathbb{Z}_{41}$.

ii. Give me some meaning to (i).

ans. $17^{41} = 17$ in $\mathbb{Z}_{41}$

This also means that $\quad 17^{41} \equiv 17 \pmod{41}$

and also $\quad 41 \mid 17^{41} - 17$: i.e 41 is a factor of $17^{41} - 17$

or even $\quad 17^{41} = q41 + 17$ for some $q \in \mathbb{Z}$

iii. Assume that $\gcd(a, 15) = 1$. Convince me that $a^{27} \pmod{15} = a^3 \pmod{15}$

ans: assume $n = 15$.

then $\phi(n) = 8$

By Euler – Fermat Result, $a^8 \pmod{15} = 1$, for any $a \in N^*$

now, $a^{27} = a^{8+8+8+3}$

$= a^8 \cdot a^8 \cdot a^8 \cdot a^3$

$a^{27} \pmod{15} = a^8 a^8 a^8 a^3 \pmod{15}$

Recall $xy \pmod{n} = x \pmod{n} \cdot y \pmod{n}$

Hence $a^{27} \pmod{15} = a^8 \pmod{15} \cdot a^8 \pmod{15} \cdot a^8 \pmod{15} \cdot a^3 \pmod{15}$

$= 1 \times 1 \times 1 \times a^3 \pmod{15}$

$\therefore$ $a^{27} \pmod{15} = a^3 \pmod{15}$

iv. Convince me that $2^{165} \pmod{15} = 2$. (note that $\phi(15) = 8$ and $165 = 8 \times 20 + 5$ and $\gcd(2,15) = 1$)

ans: let $a = 2$ and $n = 15$,

$\gcd(2,15) = 1$. and $\phi(15) = 8$

So by Euler – Fermat result, $2^8 \equiv 1 \pmod{15}$

now, $2^{165} = 2^{8 \times 20} 2^5$ ~~mod~~

$2^{165} \pmod{15} = 2^{8 \times 20} \pmod{15} \cdot 2^5 \pmod{15}$

$= 1 \times 2^5 \pmod{15}$

$= 2.$

DATE

Q5. Use Induction to prove $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ for every $n \in \mathbb{N}^*$

① Prove it for $n=1$

$$\sum_{i=1}^{1} \frac{1}{i(i+1)}\,(i=1) = \frac{1}{(1+1)} = \frac{1}{2} \checkmark$$

② Assume it is true for $n = k \geq 1$

$$\sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{(k+1)} \text{ is true} \checkmark$$

③ Prove it for $n = k+1$

we need to show that

$$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \frac{k+1}{(k+1)+1}$$

$$= \sum_{i=1}^{k} \frac{k}{(k+1)} + \frac{1}{(k+1)(k+1+1)}$$

$$= \frac{k(k+2)+1}{(k+1)(k+2)}$$

$$= \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2} = \frac{k+1}{(k+1)+1} \checkmark$$

Q6. $x_1 = 4$, $x_{n+1} = \sqrt{3+4x_n}$

Use Math. Induction to prove that $x_n \leq 5$ for every $n \geq 1$

① Prove for $n=1$

$$x_{1+1} = \sqrt{3+4x_1} = \sqrt{3+4(4)}$$

$$= \sqrt{19} \leq 5 \checkmark$$

② Assume it is true for $n = k \geq 1$

$$x_k = \sqrt{3+4x_{k-1}} \leq 5$$

③ Prove it for $n = k+1$,

$$x_{k+1} = \sqrt{3+4x_k} \leq \sqrt{3+4(5)}$$

$$\leq \sqrt{23} < 5$$

Q7. (i) $\exists x \in \mathbb{N}^*$ and $\exists y \in \mathbb{Z}$ s.t. $x+y=0$

True $\quad 1 + (-1) = 0$

$2 + (-2) = 0$

$\vdots$

(ii) $\exists x \in \mathbb{N}^*$ s.t. $x+y=0$ $\forall y \in \mathbb{Z}$

False $\quad 1 + (-2) = -1 \neq 0$

$1 + 2 = 3 \neq 0$

(iii) $\exists x \in \mathbb{Z}^*$ s.t $x+y=0$ $\forall y \in \mathbb{Z}$

False $\quad 2+0 = 2 \neq 0$

HW-8

Sionell Tom

$F = \downarrow$ $\quad$ $G = \downarrow$

1.Use truth table to convince me that [$(S1 \Rightarrow S2) \vee (S1 \Rightarrow S3)$] $\equiv$ [$S1 \Rightarrow (S2 \vee S3)$] [Hint: You may use 1 for T and 0 for F, since most of you are engineers, note each si has two possibilities and since we have three statements, we will have 23 possibilities. In order to get all possibilities do the following: Under S1 write 4 consecutive 1's (T) followed by 4 consecutive 0's(F) (i.e., 1 1 1 1 0 0 0 0, Under S2: Write 2 consecutive 1's followed by 2 consecutive 0's and so on (i.e., 1 1 0 0 1 1 0 0), under S3 alternate 1 , 0 (i.e., 1 0 1 0 1 0 1 0))

S_1	S_2	S_3	$S_1 \Rightarrow S_2$	$S_1 \Rightarrow S_3$	F	$S_2 \vee S_3$	G
1	1	1	1	1	1	1	1
1	1	0	1	0	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	0	0	1	1	1	0	1

From the truth table we can see that

$$(S_1 \Rightarrow S_2) \vee (S_1 \Rightarrow S_2) \equiv S_1 (S_2 \vee S_3)$$

2.Use the truth table to convince me that S1 ∧ ¬S2 ≡ ¬(¬S1 ∨ S2) [note ¬S is called the negation of S (so if S is T (1), then the negation of S written ¬S is F(0))

S_1	S_2	$\bar{S_1}$	$\bar{S_2}$	$S_1 \wedge \bar{S_2}$	$\bar{S_1} \vee S_2$	$\overline{\bar{S_1} \vee S_2}$
1	1	0	0	0	1	0
1	0	0	1	1	0	1
0	1	1	0	0	1	0
0	0	1	1	0	1	0

From the truth table we can see that

$$S_1 \wedge \bar{S_2} \equiv (\overline{\bar{S_1} \vee S_2})$$

Let A = {0, {0, y}, y, {6}, 6, x, ϕ}, B = {{0}, {ϕ}, {6}, {6, x}, 6, y, 23, 10, {{0}, {6, x}}}. Then Write T or F

a. {{0}, {6, x}} ∈ B. T

b. {{0}, {6, x}} ⊂ B. T

c. {ϕ} ∈ A F

d. {ϕ} ∈ B T

e. {ϕ} ⊂ B F

f. {ϕ} ⊂ A T

g. ϕ ∈ A T

h. {23, 10, y} ∈ B F

i. {23, 10, y} ⊂ B T

j. {6} ∈ A ∩ B T

k. {6} ⊂ A ∩ B. T

l. (10, x) ∈ A × B F

m. {({0, y}, 6),(y, {0})} ⊂ A × B. T

n. Find A ∩ B = {y, {6}, 6}

o. Find B − A. = {{0}, {ϕ}, {6, x}, 23, 10, {{0}, {6, x}}}

p. Find |A × B|. (i.e., find the cardinality of the cartesian product A × B)

let no. of element in A = n = 7

let no. of element in B = m = 9

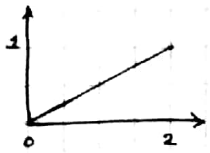
$|A \times B| = mn = 9 \times 7 = 63$

Homework :

Taneem Batool

[1] (i) Let $f : (0, 2) \to (0, 1]$ s.t. $f(x) = 0.5x$. ~~If~~ Is f a function? Is it 1-1? Is it onto? Explain briefly.

ans:



f is a function. It is 1-1 bc. every value in the co-domain is mapped back to exactly one element in the domain. f is onto bc range = co domain

Not ONTO since 1 in the range but 2not in the domain

ii) let $f : (-4, 8) \to (0, 3)$ s.t $f(x) = \sqrt{x+1}$. is f a function? is it injective? is it surjective? Explain.

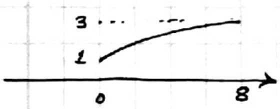
Common mistake sqrt{9} = 3 / not 3 or -3 !, so graph only upper half

ans.

f is not a function bc for values $x < -1$, there is no corresponding value in the codomain ($x < -1$ has no image). And the elements in the domain maps to more than one element in the codomain (one to many)

iii) Let $f : (0,8) \to (a\ b)$ s.t $f(x) = \sqrt{x+1}$. Find a, b so that f is bijective. Then find domain and range of f^{-1}. Write down the eq^n of f^{-1}.

ans



in order for f to be ~~bijectio~~ bijective $a = 1$ and $b = 3$.

domain of $f^{-1} = (1, 3)$.
range of $f^{-1} = (0, 8)$.

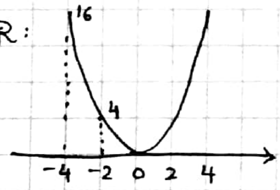
$f^{-1} \Rightarrow x = \sqrt{y+1}$

$x^2 - 1 = y$

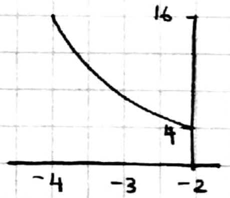
$\therefore f^{-1}(x) = x^2 - 1$

iv) Let f: ~~A~~ $(-4, b) \to (a, 4)$ s.t $f(x) = x^2$. Find a, b so that f is bijective. Then find the domain and range of f^{-1}. Write down the eqn for f^{-1}.

ans: Assuming $\mathbb{R} \to \mathbb{R}$:



To make f bijective, $a = 16$, $b = -2$:



Domain of f^{-1} : $(16, 4)$
Range of f^{-1} : $(-4, -2)$

f^{-1} : $y = x^2$

$x = y^2$

$y = -\sqrt{x}$

$\therefore f^{-1}(x) = -\sqrt{x}$

v) Let $f : \{1\ 2\ 3\ 4\ 5\ 6\ 7\} \to \{1\ 2\ 3\ 4\ 5\ 6\ 7\}$ s.t
$f(1) = 2$, $f(2) = 1$, $f(3) = 4$, $f(4) = 5$, $f(5) = 6$, $f(6) = 7$ and $f(7) = 3$

i.e $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 1 & 4 & 5 & 6 & 7 & 3 \end{pmatrix}$

Find f^2 and f^3. Write f as a composition of disjoint cycles, then find the smallest possible integer $n \geq 1$ s.t $f^n = I$

ans: f as disjoint cycles : $(1\ 2)(3\ 4\ 5\ 6\ 7)$

$f^2 = 1 \quad 2 \quad 5 \quad 6 \quad 7 \quad 3 \quad 4$

This is the range of f^2 = f o f, so f^2(1) = 1, f^2(2) = 3, ... f^2(7) = 4

$f^3 = 2 \quad 1 \quad 6 \quad 7 \quad 3 \quad 4 \quad 5$

This is the range of f^3 = f o f 0 f , so f^3(1) = 2, ..., f^3(7) = 5

$n = \text{LCM}(2, 5) = 10$. Hence $f^{10} = I$

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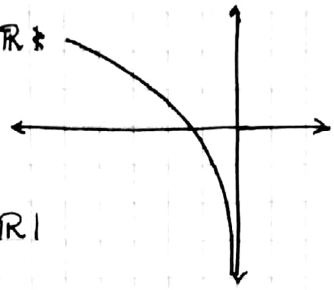
Homework 10

I i. Convince me that $|\mathbb{R}| = |(-4, 1]|$

ans: first show ~~0, ∞~~ $|(-\infty, 0]| = |\mathbb{R}|$

$f : (-\infty, 0) \to \mathbb{R}$
$f(x) = \ln(-x)$

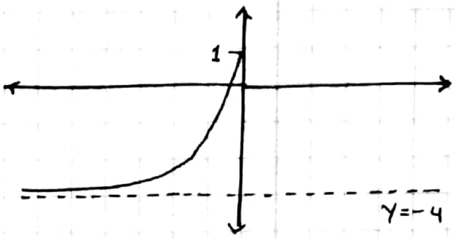


is bijective

$\therefore |(-\infty, 0)| = |\mathbb{R}|$

$|(-\infty, 0) \cup \{0\}| = |(-\infty, 0]|$
$|(-\infty, 0]| = |(-\infty, 0)| = |\mathbb{R}|$
So $|(-\infty, 0]| = |\mathbb{R}|$

second: show $|(-\infty, 0]| = |(-4, 1]|$



$g : (-\infty, 0] \to (-4, 1]$
$g(x) = 5e^{x} - 4$

is bijective

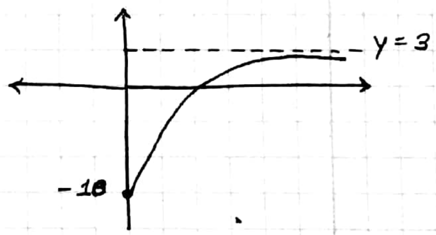
$\therefore |(-\infty, 0]| = |(-4, 1]|$ and $|(-\infty, 0]| = |\mathbb{R}|$
Hence $|\mathbb{R}| = |(-4, 1]|$

Since $\mathbb{R}$ is uncountable, $(-4, 1]$ is also uncountable

ii. Convince me that $|(-10, 3)| =$ ~~0, 00~~ $|(0, 0{\cdot}0025)|$.

ans: first: show $|(0, \infty)| = |(-10, 3)|$

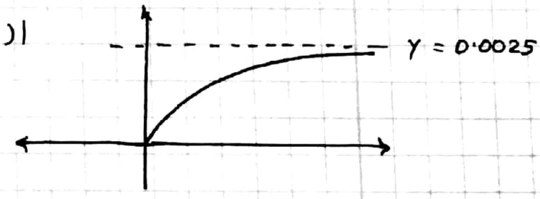
$f : (0, \infty) \to (-10, 3)$
$f(x) = -13e^{-x} + 3$



is bijective

$\therefore |(0, \infty)| = |(-10, 3)|$

Second: show $|(0, \infty)| = |(0, 0{\cdot}0025)|$



$g : (0, \infty) \to (0, 0{\cdot}0025)$
$g(x) = -0{\cdot}0025e^{-x} + 0{\cdot}0025$

is bijective

$\therefore |(0, \infty)| = |(0, 0{\cdot}0025)|$ and $|(0, \infty)| = |(-10, 3)|$
Hence $|(-10, 3)| = |(0, 0{\cdot}0025)|$

iii) Is the set $A = (0, 0{\cdot}000033) \cap \mathbb{Q}$ finite or infinite. Is A countable?

ans.

Since between any two rational numbers there are infinitely many rational numbers, we conclude that A is infinite. Now A is a subset of Q. A subset of a countable set is countable (logical...if you can count the numbers in Q, then we must be able to count any subset of Q). Hence $|A| = |N|$

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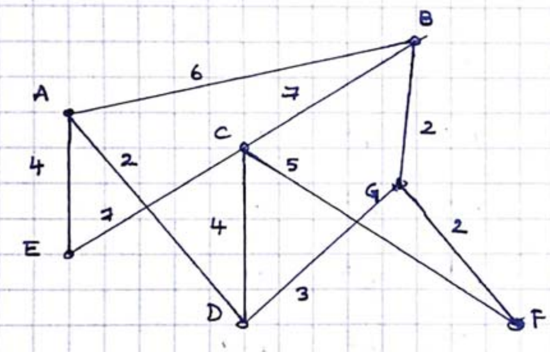
HOMEWORK 13:



Use Dijkstra's algorithm to find the minimum spanning tree.

	A	B	C	D	E	F	G
A	$\boxed{0}$	6^A	∞	2^A	4^A	∞	∞
D		6^A	6^D	$\boxed{2^A}$	4^A	∞	5^D
E		6^A	6^D		$\boxed{4^A}$	∞	5^D
G		6^A	6^D			7^G	$\boxed{5^D}$
B		$\boxed{6^A}$	6^D			7^G	
C			$\boxed{6^D}$			7^G	
F						$\boxed{7^G}$	

Assume starting from A.

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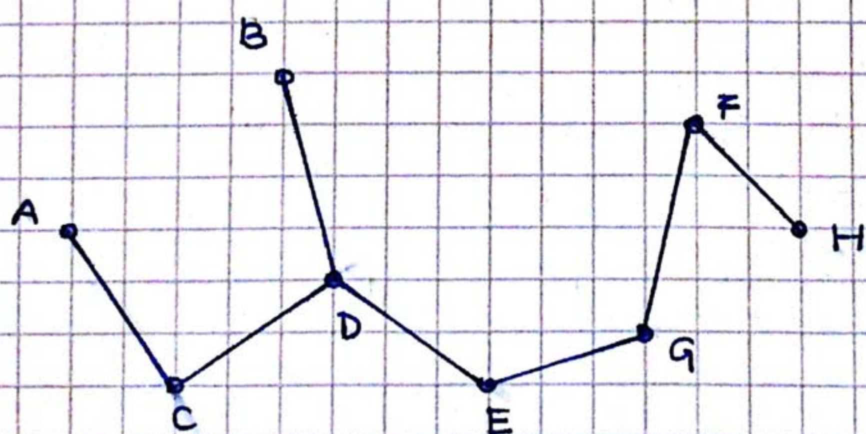
	A	B	C	D	E	F	G	H
A	$\boxed{0}$	8^A	$\underline{2^A}$	5^A	∞	∞	∞	∞
C		8^A	$\boxed{2^A}$	$\underline{4^C}$	7^C	∞	∞	∞
D		6^D		$\boxed{4^C}$	$\underline{5^D}$	10^D	7^D	∞
E		$\underline{6^D}$			$\boxed{5^D}$	10^D	6^E	∞
B		$\boxed{6^D}$				10^D	$\underline{6^E}$	∞
G						$\underline{8^G}$	$\boxed{6^E}$	12^G
F						$\boxed{8^G}$		$\underline{11^F}$
H								$\boxed{11^F}$



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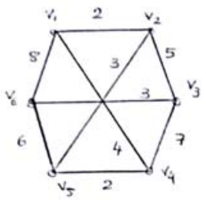


 $V_1 - V_2 - V_3 - V_4 - V_5 - V_6 - V_1 = 30$

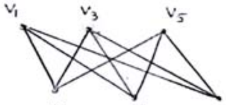
 $V_1 - V_2 - V_5 - V_4 - V_3 - V_6 - V_1 = 25$

 $V_1 - V_2 - V_3 - V_6 - V_5 - V_4 - V_1 = 22$

 $V_1 - V_4 - V_3 - V_2 - V_5 - V_6 - V_1 = 33$

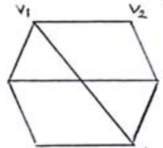
Shortest path: $V_1 - V_2 - V_3 - V_6 - V_5 - V_4 - V_1$

No odd cycles: graph is bi partite



Diameter of graph: 2 Girth = 4

Remove $V_2 - V_5$ edge :



$d(v_1, v_2) = 1$
$d(v_1, v_3) = 2$
$d(v_1, v_4) = 1$
$d(v_1, v_5) = 2$
$d(v_1, v_6) = 1$

$d(v_2, v_3) = 1$
$d(v_2, v_4) = 2$
$d(v_2, v_5) = 3$
$d(v_2, v_6) = 2$
$d(v_3, v_4) = 1$

$d(v_3, v_5) = 2$
$d(v_3, v_6) = 1$
$d(v_4, v_5) = 1$
$d(v_4, v_6) = 2$
$d(v_5, v_6) = 1$

diameter = 3.

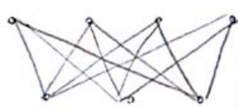
6, 4, 4, 4, 4, 2, 2, 1.
$S' = $ 3, 3, 3, 3, 1, 1, 1
$S'' = $ 2, 2, 2, 1, 1, 1
$S''' = $ 1, 1, 1, 1, 1
$S^{(4)} = $ 0, 1, 1, 1, 1
= 1, 1, 1, 0
$S^5 = $ 0, 1, 0
= 1, 0, 0
$S^6 = $ −1, 0

Not graphical.

4, 4, 4, 3, 3, 3, 3
$S' = $ 3, 3, 2, 2, 3, 3
= 3, 3, 3, 3, 2, 2
$S'' = $ 2, 2, 2, 2, 2
$S''' = $ 1, 1, 2, 2
= 2, 2, 1, 1
$S^4 = $ 1, 0, 1
= 1, 1, 0
$S^5 = $ 0, 0

x x.

Yes, is a graphical sequence



Yes, this graph can be constructed at $K_{3,4}$